PROPORTIONAL REASONING 3
ANSWER KEY

PROPORTIONAL REASONING APPLICATIONS

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 3.5.) Key mathematical vocabulary is underlined throughout the packet.

cross-multiplication property	proportion
proportional relationship	scale
scale drawing	scale factor

Lesson Notes

TWINKIE, THE DOG

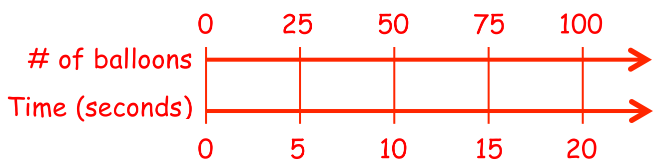
Twinkie, the Jack Russell Terrier, pops balloons. Will she break the world record? Follow your teacher's directions to learn more about this amazing dog.

Possible student notes and representations:

(1) Time estimates will vary. Some ways to represent how long it will take to break 100 balloons using a proportional reasoning model are:

a ratio of 25 to 5 or 5 : 1 or 25 balloons in 5 seconds → 100 balloons in 20 seconds

$$\frac{\text{balloons}}{\text{seconds}} \longrightarrow \frac{25}{5}\left(\frac{4}{4}\right) = \frac{100}{20}$$

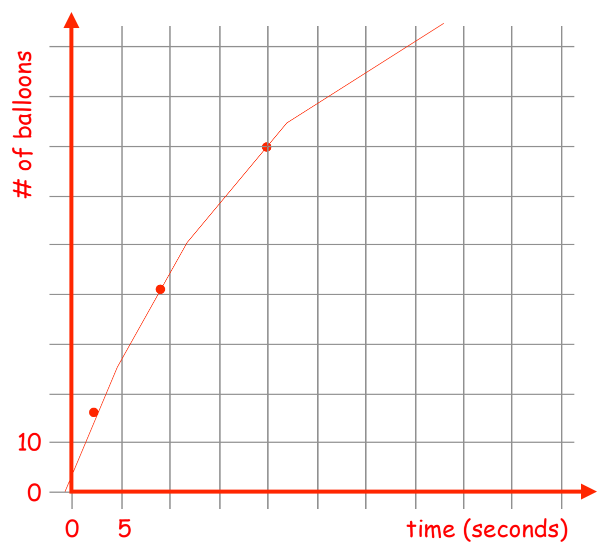


$$25 : 5 \xrightarrow{\times 4} 100 : 20$$

(2) Answers will vary. Since Cally's record is 41.67 seconds, Twinkie is on track to break the record. But once some balloons are popped, and balloons are more spread out, it may be difficult for Twinkie to keep up the pace. It is likely that she will take more than 20 seconds.

(3) and (4) Points may vary. Based on data from the video:

time (seconds) (x)	number of balloons (y)	unit rate: $\frac{\text{\# of balloons}}{\text{time inseconds}}$
5	25	5
12	50	4.17
22	75	3.41
38	100	2.63



(5) This is not a proportional relationship. Neither variable is a constant multiple of the other. The unit rates are different for different pairs of values. The graph goes though the origin, but it is not a straight line. Twinkie slowed down as the challenge progressed (she popped ewer balloons per second as she progressed).

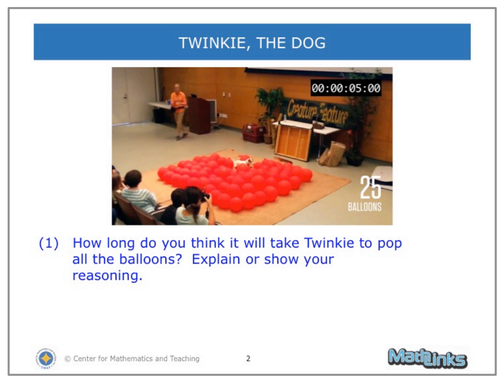
LESSON NOTES: TWINKIE, THE DOG*

On powerpoints, blue text in italics suggests discussion, text that is numbered suggests written responses.

*Go to http://www.101qs.com/3933 to use the lesson "World Record Dog" designed by Dan Meyer.

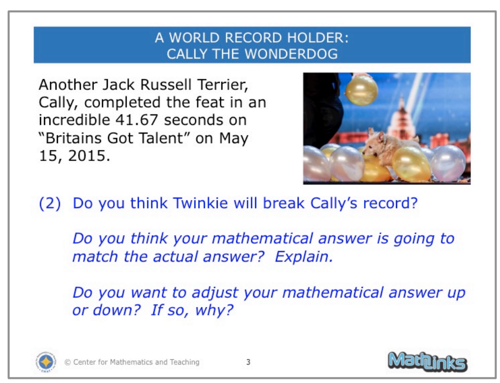
- Slide 1: Click the picture to play the video. Record students' responses to the questions. Encourage student creativity at this point. Possible student statements are: I know that there are 100 balloons. Twinkie pops 25 balloons in the first 5 sec so there are 75 balloons left. Students may ask: Can she can pop them all? How long it will take? Will she quit? Will she speed up or slow down? Will she get distracted by the kids in the audience?

- Slide 2: Explore question (1). Encourage students to guess a lower limit and upper limit for their estimates as well. Share reasoning and representations. Ask some simple questions about "popping balloons at the same rate." ***If Twinkie could pop 25 balloons every 5 seconds, then how many balloons would she pop in 10 seconds? 15 seconds? 20 seconds?*** This would illustrate a proportional relationship because the values of the ratios of $\frac{\text{balloons}}{\text{seconds}}$ are equivalent.



- Slide 3: Provide more information about the world record. Additional information about dog-popping balloon records can be researched online. (Yes, it's a thing!)

  For (2), encourage students to think about reasons why a proportional model might be useful or why it might not hold up. They may want to revise their estimates.

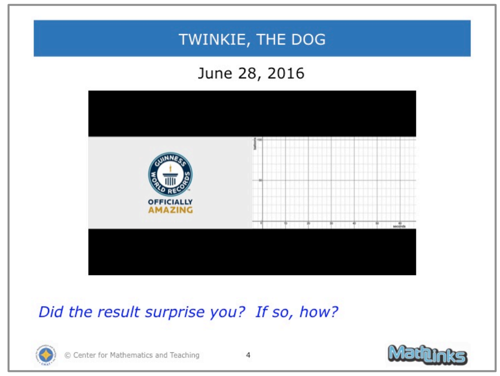


LESSON NOTES: TWINKIE, THE DOG

Continued

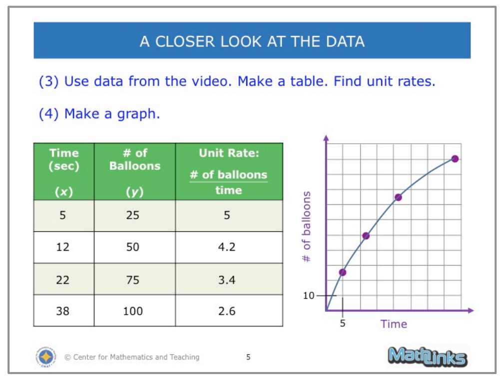
- Slide 4: Play Part 2 of the video to reveal Twinkie's record-breaking accomplishment. Encourage students to explain why results did (or did not) match their predictions.

  Replay the video several times, and stop it along the way to collect some data for time from the start, x, and # of balloons popped, y. Discuss the meaning of the graph to the right as well. ***What do the x- and y- axes represent? Why is the graph not a line? How do you know from the graph that Twinkie is slowing down as time progresses?***

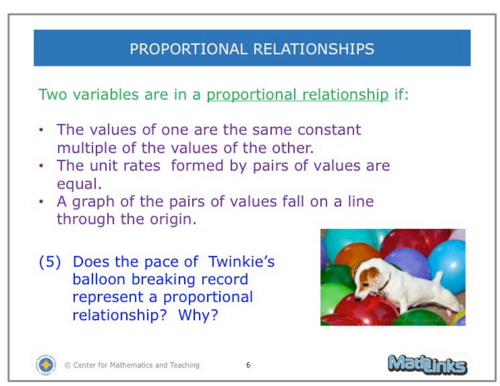


- Slide 5: Reveal (3). Ask students to make a table of Twinkie's data. Include a 3rd column on the table for unit rate at various points in time. Use the animated graphics as scaffolding to get students started or for discussion along the way. ***What do the unit rates tells us about whether Twinkie is slowing down, speeding up, or staying at the same rate throughout?*** She is slowing down since at each interval (though not "equally spaced" intervals) there are fewer balloons popped per second. ***We see 25 balloons popped in 5 seconds. How many seconds would it take to pop 50 if she kept up her pace?*** 10 sec. ***How many would she pop in 20 seconds if she kept up the initial pace?*** 100 balloons. ***If Twinkie continued to pop the same number of balloons per second (the same unit rate), then this would represent a proportional relationship.***

  Reveal (4). Ask students to make a graph of the data. ***How should we label the axes? What is an appropriate scale? Do the points fall on a line? What do you think the unit rates would be if points fell on the line?***



- Slide 6: Discuss factors that determine a proportional relationship. For (5), use the Twinkie data and graph as counter examples. The unit rates change over different intervals (she slows down). A given pair of numbers in the table generally cannot be multiplied by a constant value to obtain another pair of numbers. The graph is not a straight line.



PROPORTIONAL REASONING

We will use proportional reasoning strategies to solve problems.

GETTING STARTED

Compute.

1. \$2.56 + \$3.29 \$5.85	2. \$8.23 – \$4.68 \$3.55	3. 4 • (\$1.57) \$6.28	4. $\frac{\$8.61}{7}$ \$1.23

Use your knowledge of equivalent fractions to solve for *x*:

5. $\frac{2}{9} = \frac{6}{x}$ $\frac{2}{9} \bullet \frac{3}{3} = \frac{6}{\boxed{27}}$ $x = 27$	6. $\frac{30}{36} = \frac{x}{6}$ $\frac{30}{36} \div \frac{6}{6} = \frac{\boxed{5}}{6}$ $x = 5$	7. $\frac{x}{5} = \frac{12}{20}$ $\frac{\boxed{3}}{5} = \frac{12}{20} \div \frac{4}{4}$ $x = 3$

Find the cost. Try using sense-making strategies. See section 3.5 for ideas as needed.

8. If one pencil costs 35¢, what is the cost of 4 pencils? One way (doubling): If one pencil costs 35¢, then two pencils cost 70¢, and 4 pencils cost 140¢, or \$1.40	9. If 6 pencils cost \$2.40, what is the cost of 1 pencil? One way (find unit price): \$2.40 ÷ 6 = \$0.40

ART SUPPLIES

Lesson Notes

Mrs. Carter is buying art supplies. Help her determine the cost and quantities for some items she needs. Assume costs and quantities are in a proportional relationship. Find this phrase in section 3.5 and record its definition in the Word bank. Then follow your teacher's directions.

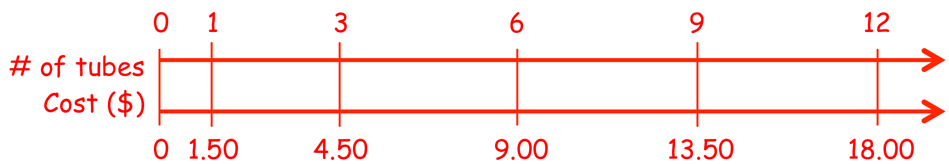
Possible responses, strategies, and notations are illustrated here.

(1) Table: Explanations will vary. Computations in the table show one way to obtain results.

# of pencils	4	12 (4×3)	2 $(4 \div 2)$	10 $(4 + 4 + 2)$	50 (10×5)
cost of pencils (in dollars)	5	15 (5×3)	2.50 $(5 \div 2)$	12.50 $(5 + 5 + 2.5)$	62.50 $(\$12.50 \times 5)$

Unit price: $\frac{\text{cost}}{\text{pencil}} \longrightarrow \frac{\$5}{4} = \$1.25$. Multiply unit price by the number of pencils to obtain cost.

(2) Double number line: This diagram shows that 12 tubes cost \$18.00 and 1 tube cost \$1.50.

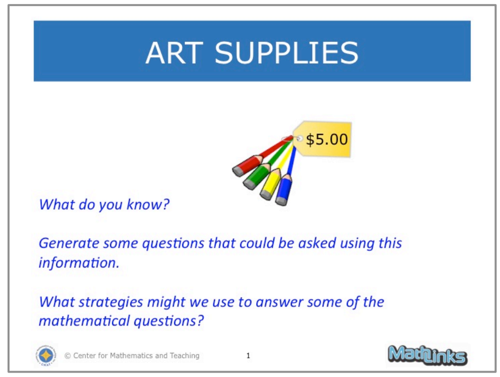


Equivalent fractions: To find the cost of 54 tubes, $\frac{\text{tubes}}{\text{cost}} \longrightarrow \frac{9}{13.50}\left(\frac{6}{6}\right) = \frac{54}{81}$

Unit price: To find the cost of 100 tubes, $\frac{\text{cost}}{\text{tube}} \longrightarrow \frac{1.50}{1}(100) = \$150$

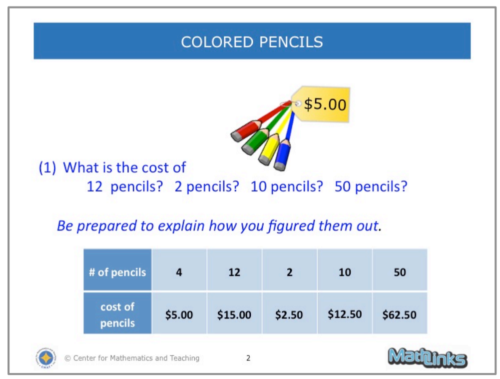
LESSON NOTES: ART SUPPLIES

- Slide 1: Begin with the picture. Discuss what we know (4 pencils cost $5), and questions that could be asked using this information. Record student ideas. ***If we assume a proportional (multiplicative) relationship between number of pencils and cost, then what are some strategies we might use to find the cost of different numbers of pencils?*** Strategies include: make a table; make a double number line or tape diagram; draw a graph, use equivalent fractions (or arrow diagrams); multiply unit rates by the number of pencils to be purchased. Some students may know how to solve an equation (proportion). This is introduced later in this lesson.



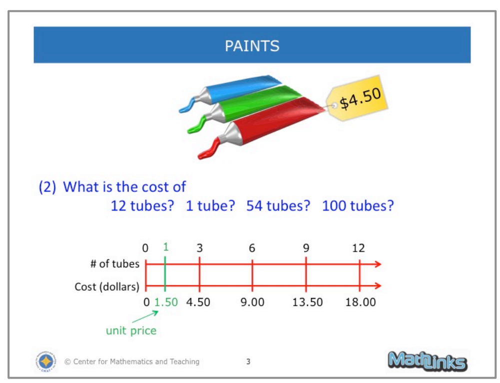
- Slide 2: For (1), give students time to find the cost of different numbers of pencils. Discuss strategies. ***Are some strategies more efficient when we need to solve multiple problems using the same information?*** Students may observe that a double number line or table works well here. They may also want to try making equivalent fractions or using a unit price.

  If students are struggling with a table, reveal the steps of the animated table on the slide and ask students to explain how entries could be computed. For example, to find the cost of 10 pencils, multiply the cost of 2 pencils by 5.



- Slide 3: For (2), repeat the exploration process for the tubes of paint problem. Encourage students to use a different representation than they used for the colored pencil problem. ***If we did not assume a proportional relationship between tubes of paint and cost, why could we not answer the questions for (2)?*** Without this assumption, we would not have a multiplier to find different costs.

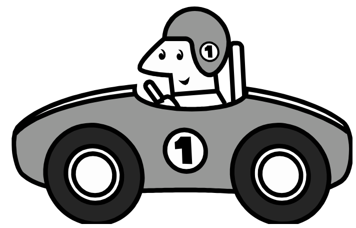
  If students struggle with a double number line, reveal steps of the animated diagram and ask students to explain how entries could be computed. Discuss limitations (we need to extrapolate past what is given on the double number line shown to find costs for 54 or 100 tubes), and so other strategies (equivalent fractions or unit prices) may be useful.



PRACTICE 1

Solve these problems using the given strategy. Check your work using another strategy of your choice.

1. Use a table.



Jake's car used 15 gallons of gas to travel 330 miles.

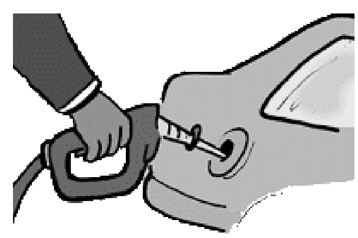
a. If the car continues to consume gas at this same rate, how far can the car go on 20 gallons? 440 miles

b. How many miles per gallon does the car get?
22 miles per gallon

Table values may vary.

Number of Gallons	Number of Miles
15	330
5	110
1	22
20	440

2. Use a double number line.

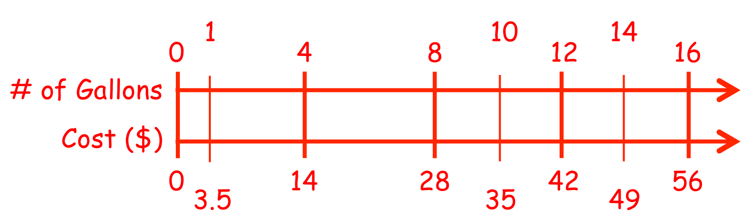


Angie paid $14 for 4 gallons of gas.

a. At this rate, how many gallons of gas can she buy for $35?
10 gallons

b. How much will 14 gallons of gas cost? $49

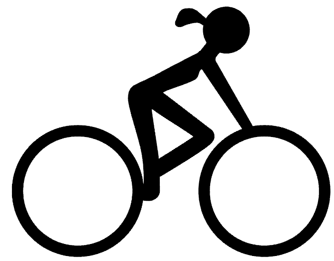
c. What is the price per gallon? $3.50



PRACTICE 2

Solve these problems using the strategy given. Check your work using another strategy of your choice.

1. Use a table or a double number line.



Samara biked 6 miles in 30 minutes.

a. At that rate, how far could she go in 2 hours? 24 miles

b. At that rate, how far could she go in 1 hour? 12 miles

c. At that rate, how long would it take her to go 15 miles? 75 minutes (1.25 hr)

Table is shown. Students may choose instead to use a double number line.

# of Miles	6	12	24	1	10	5	15
# of Minutes	30	60	120	5	50	25	75

2. Use some form of arithmetic, such as a unit rate or a chunking strategy.

Greg is training for a marathon. He ran 21 miles in $3\frac{1}{2}$ hours.

a. At that pace, how far did he run in one hour? 6 miles

b. At that pace, *about* how long will it take him to run the marathon (26.2 miles)?
At that pace Greg can run 27 miles in 4.5 hours, so it should take him a little less than 4.5 hours to run 26.2 miles. (Approximately 4 hours, 22 minutes.)

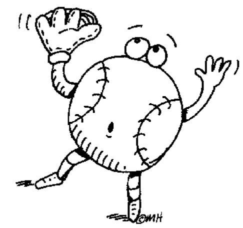
Unit rate: $\frac{\text{miles}}{\text{hours}} \longrightarrow \frac{21}{3.5} = 6$ miles per hour.

21 miles in 3.5 hrs → plus another 6 miles in 1 hour → 27 miles in 4.5 hrs.

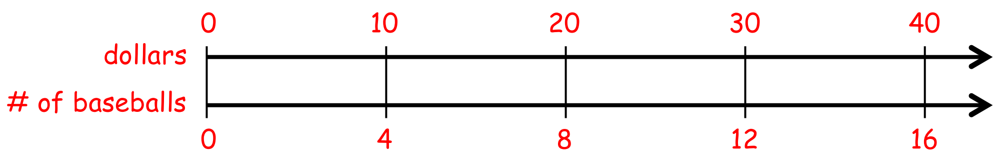
PROPERTIES OF PROPORTIONS

Lesson Notes

Follow your teacher's directions to learn about proportions. Find this word in section 3.5 and record its definition in the Word bank.



(1)
Four baseballs cost $_____.



(2)

If $\frac{20}{8} = \frac{30}{12}$ then $\frac{8}{20} = \frac{12}{30}$.

- We can create proportions by comparing dollars to baseballs OR baseballs to dollars.
- If a proportion is true, then we can "flip" both fractions, the new fractions will be equivalent, and the equation will still be true. We sometimes refer to this as the "fraction-inverse property for proportions."

Another example: If $\frac{3}{4} = \frac{6}{8}$ then $\frac{4}{3} = \frac{8}{6}$.

(3)

If $\frac{20}{8} = \frac{40}{16}$ then $\frac{20}{40} = \frac{8}{16}$.

- We can create proportions by forming equivalent ratios of quantities that are aligned either vertically or horizontally in the double number line.
- Another way to say this is that we can create equivalent ratios by comparing rates or comparing like units.
- If a proportion is true, fractions created from corresponding numerators and denominators will be equivalent.

Another example: If $\frac{3}{4} = \frac{6}{8}$ then $\frac{3}{6} = \frac{4}{8}$.

(4)

If $\frac{10}{4} = \frac{30}{12}$ then $10 \bullet 12 = 30 \bullet 4$.

- If a proportion is true, then the product of the diagonals will be equal.
- We will call it the "cross-multiplication property for proportions."

Another example:

If $\frac{3}{4} = \frac{6}{8}$ then $3 \bullet 8 = 4 \bullet 6$.

(5)

Solving a proportion using the cross-multiplication property:

$$\frac{10}{4} = \frac{x}{9}$$

$$\frac{10 \bullet 9}{4} = \frac{4x}{4}$$

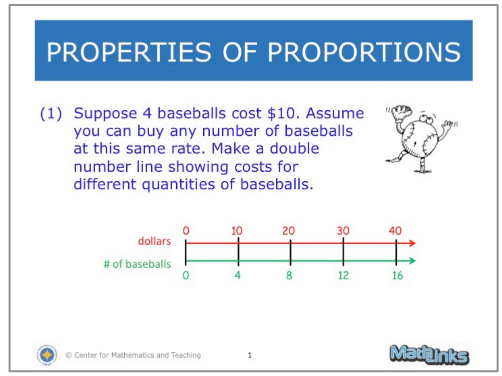
$$22.5 = x$$

9 baseballs will cost $22.50

LESSON NOTES: PROPERTIES OF PROPORTIONS

This set of animated slides aims to promote discussions about important properties of proportions and a well-known strategy for solving them. Allow time for students to digest and discuss each animated step as it is revealed. Student's written responses will summarize the discussions.

- Slide 1: For (1), ask students to create a double number line using the fact statement (Four baseballs cost $10), and then reveal the possible double number line provided.



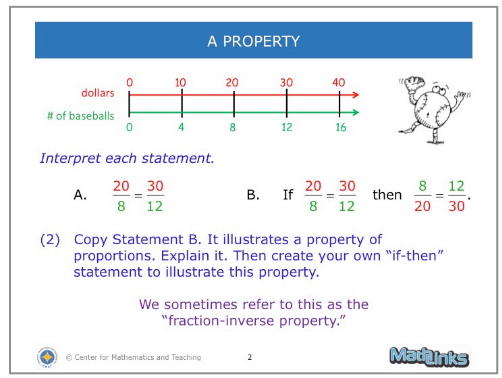
- Slide 2: Ask students to interpret the statements. Possibilities include:

  A. $20 for 8 baseballs is the same rate as $30 for 12 baseballs. $\frac{20}{8}$ = 2.5 (or $2.50 per ball). $\frac{30}{12}$ = 2.5

  B. If $20 for 8 baseballs is the same rate as $30 for 12 baseballs, then 8 baseballs for $20 is the same rate as 12 baseballs for $30. $\frac{8}{20} = \frac{2}{5}$ and $\frac{12}{30} = \frac{2}{5}$

  For (2), see answer key. Share student explanations and examples.

  We refer to this as the "fraction-inverse property for proportions," although informal names created by students, such as the "flip property," are fine.

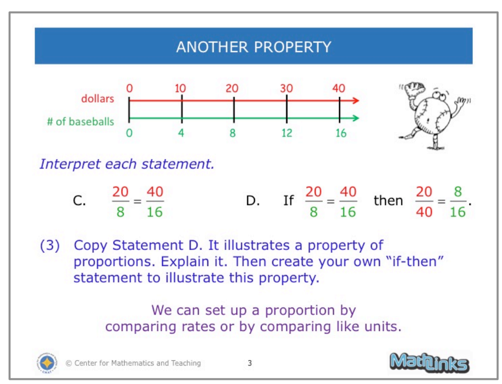


- Slide 3: Ask students to interpret each statement. Possibilities include:

  C. $20 for 8 baseballs is the same rate as $40 for 16 baseballs. $\frac{20}{8}$ = 2.5 (or $2.50 per ball). $\frac{40}{16}$ = 2.5

  D. When we compare ratios of corresponding numbers aligned vertically (rates/between units), the result is a true proportion. When we compare ratios of corresponding numbers aligned horizontally (like units/ within units), the result is also a true proportion.

  For (3), see answer key. Share student explanations and examples.



LESSON NOTES: PROPERTIES OF PROPORTIONS

Continued

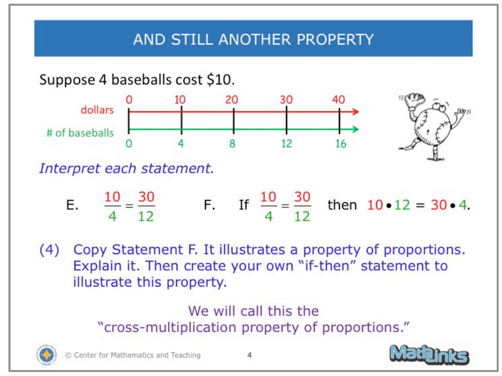
- Slide 4: Ask students to interpret each statement. Possibilities include:

  E. \$10 for 4 baseballs is the same rate as \$30 for 12 baseballs. $\frac{10}{4}$ = 2.5 (or \$2.50 per ball). $\frac{30}{12}$ = 2.5

  F. Products of diagonals in a proportion are equal.

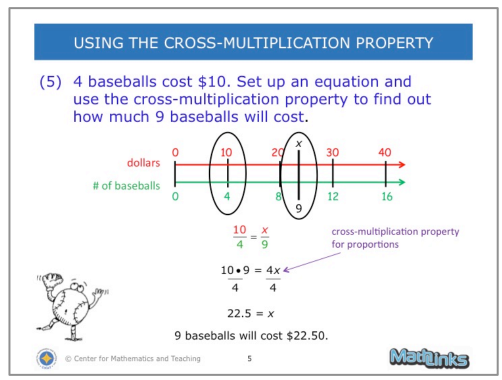
  For 4, see answer key. Share student explanations and examples.

  This is the well-known "cross-multiplication property for proportions." Have students record this in the Word Bank.



- Slide 5: This slide shows students how to use a double number line to create a proportion, and then apply the cross-multiplication property for proportions to solve it.

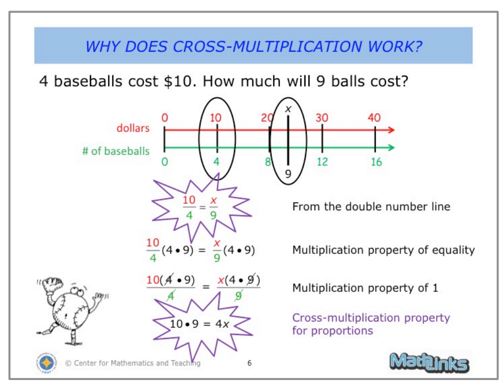
  For (5), ask students to try the procedure. Guide them slowly through the sequence of steps, asking clarifying questions along the way.



- Slide 6: The cross-multiplication property for proportions is a valid shortcut because it bypasses several steps for solving this kind of equation. This slide shows students why this property works.

  Guide students slowly through the sequence of steps so that they are reminded about how to solve this type of equation, and they see why the property works.

  Most students will prefer to apply the cross-multiplication property for proportions over standard equations solving procedures. But it is important that students know that this shortcut is based on fundamental mathematical principles. It is not simply a trick or senseless procedure.



PRACTICE 3

Use equivalent fractions or the cross-multiplication property to solve each equation.

1. $\frac{2}{5} = \frac{x}{20}$ $x = 8$	2. $\frac{3}{55} = \frac{x}{55}$ $x = 3$	3. $\frac{137}{5} = \frac{x}{55}$ $x = 1507$
4. $\frac{2}{x} = \frac{3}{13}$ $x = 8.6666... = 8.\overline{6} \approx 8.7$ or $x = \frac{26}{3} = 8\frac{2}{3}$	5. $\frac{1}{2} = \frac{5}{x}$ $x = 10$	6. $\frac{2.5}{5} = \frac{x}{12}$ $x = 6$

7. Some students explored the equation $\frac{3}{5} = \frac{6}{10}$, and rewrote it in a few different ways.

Circle the two true equations. For the equation that is not true, explain to that student why it is not true and how to revise his work.

Abner: $\frac{3}{6} = \frac{5}{10}$ Mick: $\frac{6}{3} = \frac{5}{10}$ Buck: $\frac{5}{3} = \frac{10}{6}$

Mick, if you use the cross-multiplication property, you'll see that $6 \bullet 10 \neq 5 \bullet 3$. If you want to take the original proportion equation and create a fraction by comparing the numerators ($\frac{6}{3}$), then the equivalent fraction will compare denominators in the same order ($\frac{10}{5}$). A correct statement ($\frac{6}{3} = \frac{10}{5}$) is the inverse of Abner's statement.

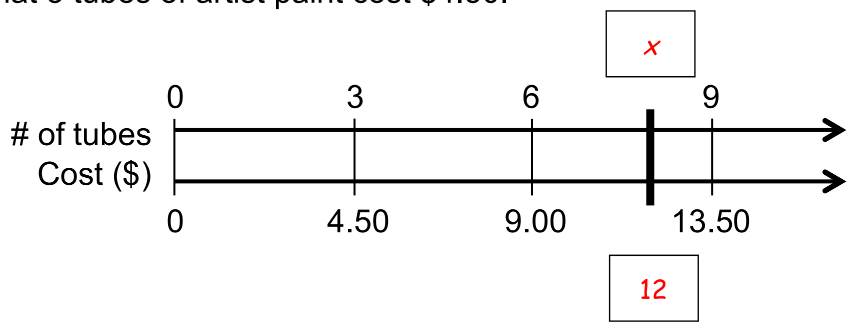
8. Rewrite the equation $\frac{2}{7} = \frac{6}{21}$ in three other ways to create true equations. Some possibilities:

$\frac{7}{2} = \frac{21}{6}$	$\frac{7}{21} = \frac{2}{6}$	$\frac{21}{7} = \frac{6}{2}$

ART SUPPLIES - REVISITED

Use a double number line to help you set up proportions and solve problems.

Recall that 3 tubes of artist paint cost $4.50.



Proportion equations may vary for the problems below, but the solutions may not.

1. How many tubes can you buy for $12?

   a. Fill in the boxes above to indicate 12 dollars and x tubes.

   b. Write a proportion and solve it. Then answer the question.

$$\frac{\#\text{ tubes}}{\$} \rightarrow \frac{6}{9} = \frac{x}{12}$$

$$9x = 72$$

$$x = 8$$

You can buy 8 tubes of paint for $12.

2. What is the cost of 50 tubes of paint?

$$\frac{\#\text{ tubes}}{\$} \rightarrow \frac{6}{9} = \frac{50}{x}$$

$$6x = 450$$

$$x = 75$$

It costs $75 for 12 tubes of paint.

3. How many tubes of paint can you buy for $42?

$$\frac{\#\text{ tubes}}{\$} \rightarrow \frac{6}{9} = \frac{x}{42}$$

$$9x = 252$$

$$x = 28$$

You can buy 28 tubes of paint for $42.

4. What is the unit price for a tube of paint?

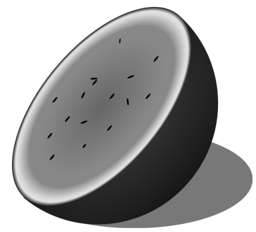
$$\frac{\$}{\#\text{ tubes}} \rightarrow \frac{4.50}{3} = 1.50$$

$1.50 per tube

PRACTICE 4

Solve using strategies of your choice (tables, unit prices, double number lines, equivalent fractions, proportions). See section 3.5 for different strategies. Strategies will vary.

1. At the Green Grocer, 2 melons cost \$3.50, and you can purchase as many as you want at this same rate.



   a. What is the cost per melon? \$1.75 per melon

   b. How much will 42 melons cost? \$73.50

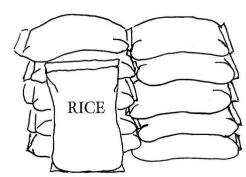
   c. How many melons can be purchased for \$84? 48 melons

a. $\frac{\$}{\text{melons}} \rightarrow \frac{3.50}{2} = 1.75$

b. $42 \bullet 1.75 = 73.50$

c. $\frac{\$}{\text{melons}} \rightarrow \frac{3.50}{2} = \frac{84}{x} \rightarrow x = 48$

2. At the Grain Grocer, 5 pounds of rice cost \$4.00, and you can purchase any amount at this rate.



   a. What is the cost per pound? \$0.80 per pound

   b. How many pounds can be purchased for \$1.00? 1.25 pounds

   c. Carlos needs two pounds of rice for a casserole. How much will that cost? \$1.60

a. $\frac{\$}{\text{lb}} \rightarrow \frac{4.00}{5} = 0.8$

b. $\frac{\$}{\text{lb}} \rightarrow \frac{0.80}{1} = \frac{1.00}{x} \rightarrow x = 1.25$

c. $2 \bullet 0.80 = 1.60$

BEST BUY PROBLEMS

We will use tables, graphs, and equations to learn more about the behavior of proportional relationships.

GETTING STARTED

Circle the better buy for each situation below and explain your reasoning. No calculations are necessary.

1. 0.75 pounds of oranges for $1.00 or 1.25 pounds of oranges for $1.00 More pounds for the same cost.	2. 3 pounds of bananas for $3.65 or 3 pounds of bananas for $4.15 Less money for the same amount.

Suppose you are running out of your favorite energy snacks, so you compare prices at two stores before making a purchase.

BARTER JACK'S

Healthy Crunch: 2 for $2.50
Super Bar: 3 for $3.25

QUIGLEY'S

Healthy Crunch: 2 for $2.75
Super Bar: 4 for $3.25

3. Without doing any calculations, explain which store offers the better buy for Healthy Crunch. At Barter Jack's, the same amount is cheaper.	4. Without doing any calculations, explain which store offers the better buy for Super Bar. At Quigley's, you get more for the same price.

Lesson Notes

SOCKS

Follow your teacher's directions to explore ways to represent which store has the better buy.

(1)

SOX 'R US	CRAZY SOCKS
2 pairs for $3	5 pairs for $6

Not all relationships are linear, so lines can be drawn here to show a "linear trend."

(2) At Sox 'R Us, one pair costs $1.50 $\left(\frac{3.00}{2} = 1.5\right)$.

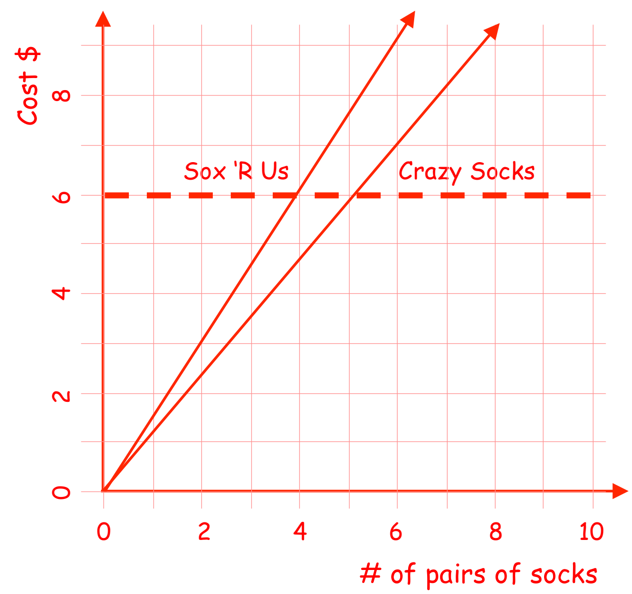
At Crazy Socks, one pair costs $1.20 $\left(\frac{6.00}{5} = 1.2\right)$.

Therefore, Crazy Socks is the better buy.

(3) Possibilities include tables, graphs, and equations.

Sox 'R Us	
# of pairs (x)	Cost in $ (y)
2	3
4	6
6	9
8	12
10	15
1	1.5
Equation: $y = 1.5x$	

Crazy Socks	
# of pairs (x)	Cost in $ (y)
5	6
10	12
15	18
20	24
25	30
1	1.2
Equation $y = 1.2x$	



- The table has several entries that allow us to compare. The circled entries show that we pay less money for the same number of socks at Crazy Socks. We might also notice that for $6, we get 4 pairs at Sox 'R Us, but we get 5 pairs at Crazy Socks. Therefore Crazy Socks is the better buy.
- The graph shows trend lines (discrete points are appropriate too). The dotted line focuses attention on the quantities you can buy for $6.00. You can buy 4 pairs at Sox 'R Us and 5 pairs at Crazy Socks.
- Finding the cost for 1 pair (unit price) in the table also gives a clue to the equations. This represents the point (1, k). The equations (in the form y = kx) tell us that Crazy Socks is cheaper as well.

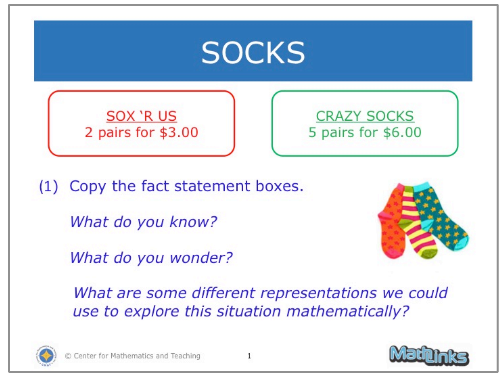
LESSON NOTES: SOCKS

For this context, we will assume a proportional (multiplicative) relationship. That is, we are comparing variables where one is a constant multiple of the other.

- Slide 1: For (1), ask students to record cost and quantity information in the boxes. Then discuss what this means, questions we might ask, and representations we might use to explore this context.

  Students may wonder: Are they the same quality? Who has better designs? What is the cost per pair? Can you buy any number of pairs of socks at this rate (i.e., a proportional relationship)? Which has the better buy on socks?

  Possible representations include unit rate computations, tables, graphs, and equations.

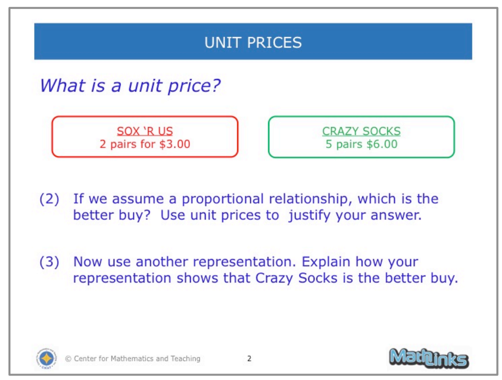


- Slide 2: Discuss the meanings of <u>unit price</u> (unit rate) and <u>proportional relationship</u>. Both are defined in section 3.5.

  For (2), unit price is a straight forward computation for determining the better buy, although students have been using other strategies in previous lessons.

  For (3), allow time for students to determine the better buy with different representations. This allows students to explore and practice other representations that give more insight into how proportional relationships behave.

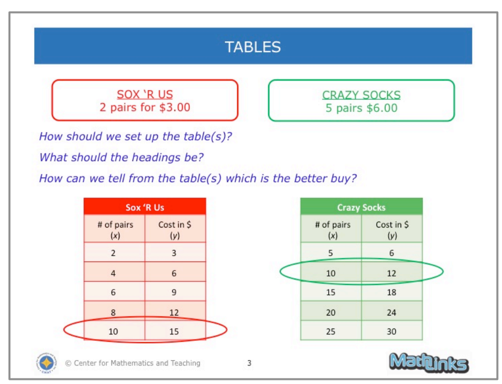
  Use slides 3, 4, 5, and 6 for scaffolding and hints as needed. They are not intended to be comprehensive.



- Slide 3: Use questions to help students get started creating tables.

  How many tables do we need? 2 ***How can we tell from the tables, which is the better buy?*** There are several entry pairs that shed light on this question. The circled entries show a greater cost for 10 pairs of socks at Sox 'R Us than at Crazy Socks. The table also shows that you can purchase more socks at Crazy Socks for $12 (or $6) than at Sox 'R Us.

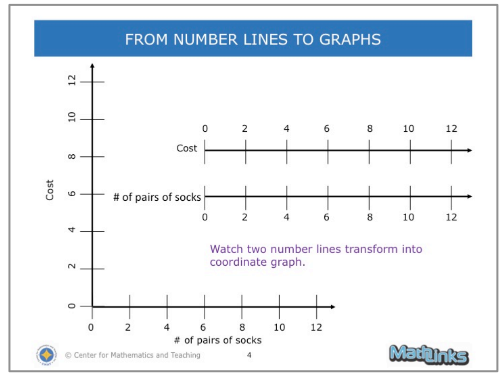
  What would the point (0,0) represent on these tables? You can buy 0 pairs of socks for $0.



LESSON NOTES: SOCKS

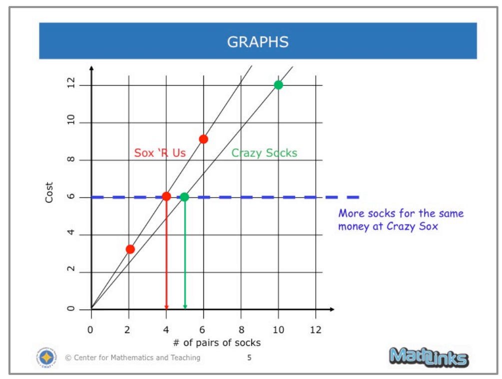
(Continued)

- Slide 4: This animated slide makes the connection between data represented on two number lines (or a double number line) and a coordinate plane. ***Does it matter which line becomes the x-axis and which becomes the y-axis?*** Technically, no. But some variables have a traditional home. Typically cost goes on the *y*-axis. Time (not in this example) typically goes on the *x*-axis.

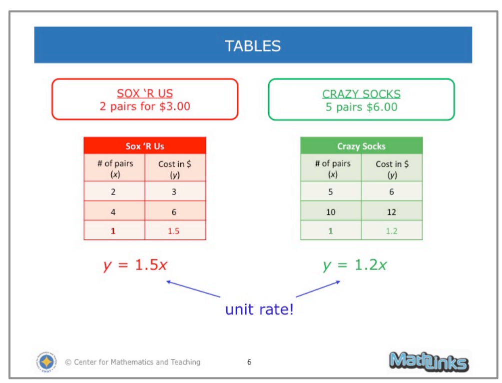


- Slide 5: Help students create an appropriate scale and graph coordinates from their tables if needed. ***Should our graphs be lines or discrete points?*** Points best represent this situation. But if the variables represent a proportional relationship, all points will lie on a line through the origin. Therefore, drawing a line is useful.

  What does the blue dotted line tell us? This represents \$6 for any number of pairs of socks. Since it crosses the Sox 'R Us line at $x = 4$, and the Crazy Socks line at $x = 5$, it shows that Crazy Socks is the better buy (more socks for the same price).



- Slide 6: Revisit the tables and record the cost for one pair (unit price) if it wasn't included earlier. This entry should help students find a rule for the cost, given the number of pairs of socks. Both tables suggest proportional relationships because one variable is a multiple of another. This is evident in the equations, which are in the form $y = kx$, where k is the constant of proportionality (and also the unit rate).



Why is the entry for 1 pair of socks significant? It is the cost for one pair. It is the unit rate. The point $(1, k)$ on the graph also tells us the unit rate. In the equation, k is the slope of the line, which will be studied when linear functions are tackled.

TORTILLAS

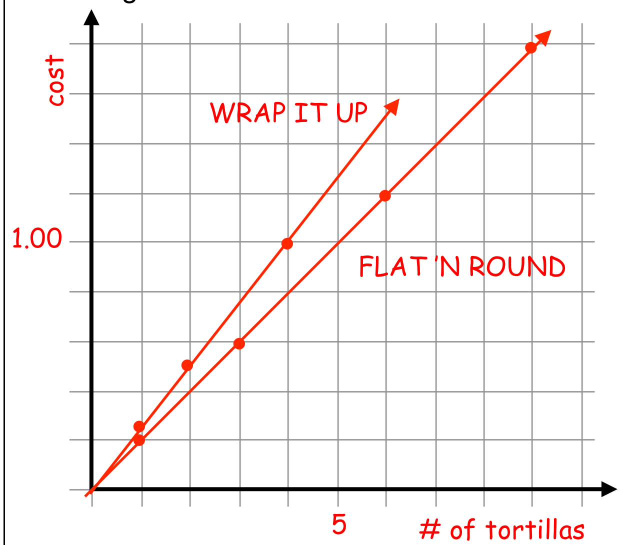
FLAT 'N ROUND	WRAP IT UP
3 tortillas for \$0.60	4 tortillas for \$1.00

1. Complete the tables below. Assume each shop will sell any number of tortillas at the rates shown.

FLAT 'N ROUND	
# of tortillas (x)	cost (\$) (y)
3	0.60
6	1.20
9	1.80
2	0.40
1	0.20

WRAP IT UP	
# of tortillas (x)	cost (\$) (y)
4	1.00
8	2.00
12	3.00
2	0.50
1	0.25

2. Label and scale the grid. Graph the data using two different colors.



3. Identify the y-coordinate when $x = 1$.

FLAT 'N ROUND (1, 0.20)

WRAP IT UP (1, 0.25)

4. Write equations to relate the number of tortillas to the cost.

FLAT 'N ROUND: $y = 0.20x$

WRAP IT UP: $y = 0.25x$

5. How are the coordinates for the ordered pairs in problem 3 related to the equations problem 4? The y-coordinate is the same as k (coefficient of x). The unit price is the same as k.

6. How do you know that the point (0, 0) satisfies the equations?

Both lines go through the origin on the graph. We can test the point (0, 0) in the equation.

FLAT 'N Round: $y = 0.2x \rightarrow 0 = 0.2(0)$ ✓ WRAP IT UP: $y = 0.25x \rightarrow 0 = 0.25(0)$ ✓

Note that it makes sense that 0 tortillas should cost \$0.00.

7. Why do these graphs and equations suggest proportional relationships?

The values of the ratios (unit rates or unit prices) created by data pairs are the same.
An equation in the form $y = kx$ fits all corresponding data pairs.
Graphed data pairs fall on a line through the origin (0, 0).

PRACTICE 5

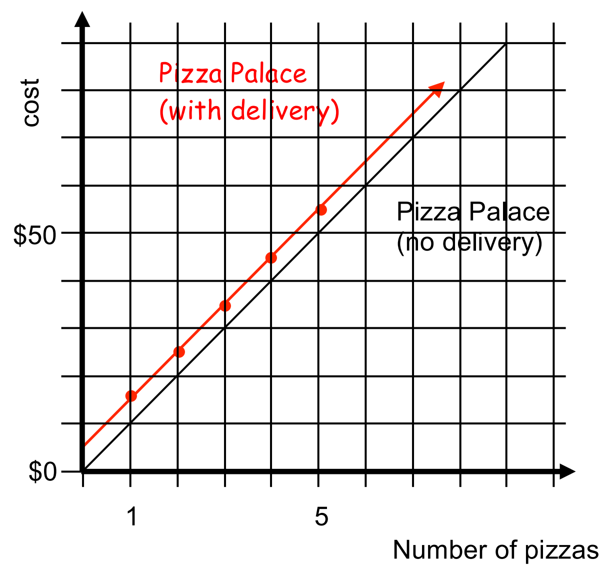
A graph for Pizza Palace prices is given. They also offer delivery for any number of pizzas for a fee of $5.00.

1. Complete the tables.

Pizza Palace (no delivery)	
# of pizzas (x)	cost \$ (y)
5	50
4	40
3	30
2	20
1	10

Pizza Palace (with delivery)	
# of pizzas (x)	cost \$ (y)
5	55
4	45
3	35
2	25
1	15

2. Graph Pizza Palace with delivery.



3. Write equations that relate cost (y) to number of pizzas (x).

Pizza Palace: $y = 10x$ Pizza Palace (with delivery): $y = 10x + 5$

4. Compare unit prices for each store. Use a calculator if needed.

no delivery	$\frac{\text{cost in dollars}}{\text{\# of pizzas}}$	$\frac{50}{5}$	$\frac{\boxed{40}}{4}$	$\frac{30}{3}$	$\frac{20}{2}$	$\frac{10}{1}$
	Unit price (in dollars/pizza)	10	10	10	10	10

with delivery	$\frac{\text{cost in dollars}}{\text{\# of pizzas}}$	$\frac{\boxed{55}}{5}$	$\frac{\boxed{45}}{4}$	$\frac{35}{3}$	$\frac{25}{2}$	$\frac{15}{1}$
	Unit price (in dollars/pizza)	11	11.25	11.67	12.50	15

5. Which of these situations represents a proportional relationship? No delivery
   Why? Unit prices for data pairs are the same. The equation $y = 10x$ is a line through the origin.

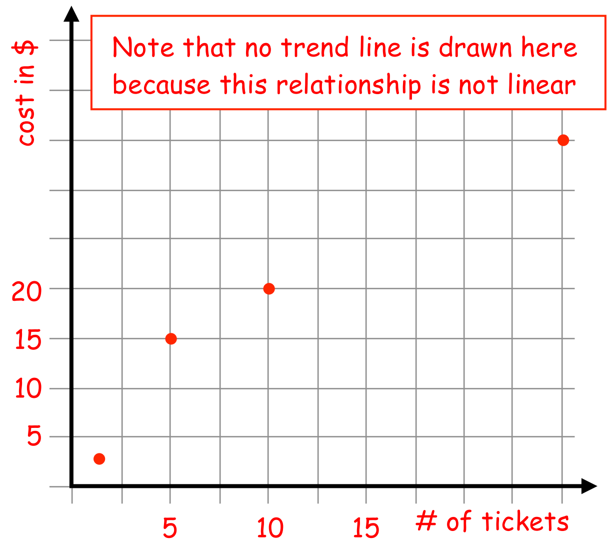
6. Which of these situations does not represent a proportional relationship? With delivery
   Why? The unit prices for data pairs are not the same. The equation $y = 10x + 5$ does not go through the origin.

PRACTICE 6

1. Here are some ticket price options at a local amusement park. Find the unit price for the different plans.

number of tickets (x)	cost \$ (y)	$\frac{\text{cost}}{\text{ticket}}$
1	3	\$3/ticket
5	15	\$3/ticket
10	20	\$2/ticket
20	30	\$1.50/ticket
30	50	\$1.67/ticket

2. Graph the relationship between number of tickets and cost. Be sure to label and scale axes appropriately.

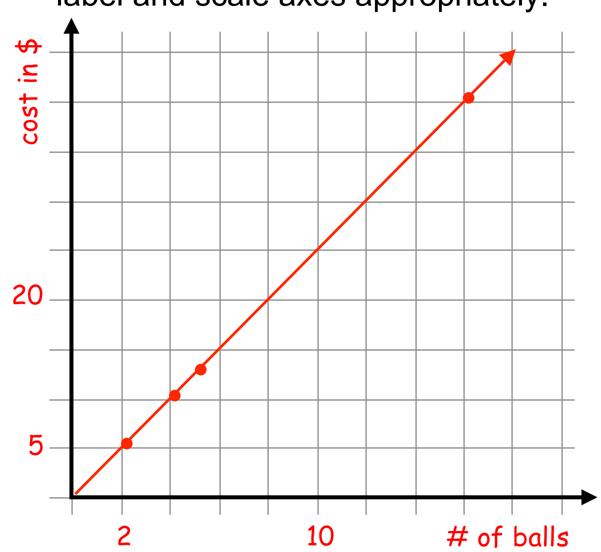


3. Does the ticket pricing represent a proportional relationship? How do you know?
   No. Unit prices are not the same for corresponding values of variables. The points, when graphed, do not fall on a line through the origin. No equation in the form $y = kx$ could represent this data.
4. Which ticket option would you choose? Why? Answers will vary.
   The best deal is 20 tickets for \$30. But it is not good to buy more tickets than needed.

5. Here are some costs and quantities for purchasing baseballs. Find the unit price for different quantities.

# of baseballs (x)	cost \$ (y)	unit price
4	10	\$2.50/ball
16	40	\$2.50/ball
2	5	\$2.50/ball
5	12.50	\$2.50/ball

6. Graph the relationship between number of baseballs and the cost. Be sure to label and scale axes appropriately.



7. Does this represent a proportional relationship? How do you know?
   Yes. Unit rates are the same. Points on the graph fall on a straight line through the origin. The line goes through the origin. An equation for the data is $y = 2.5x$.

SCALE DRAWINGS

We will make and interpret scale drawings. We will learn the meaning of scale factor and scale.

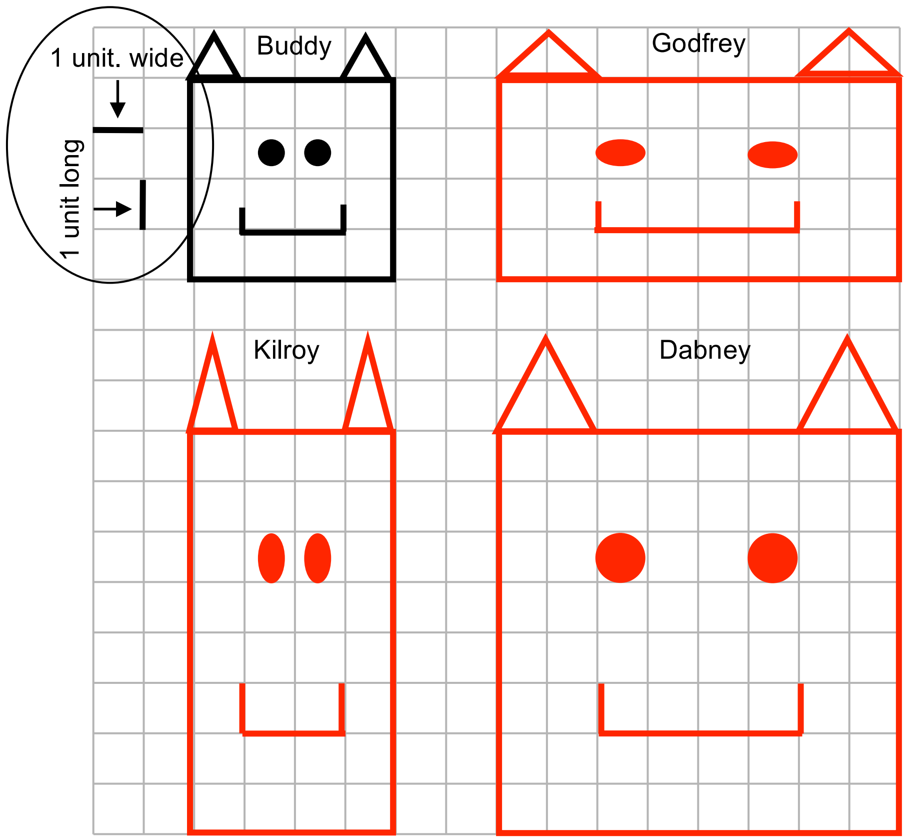
GETTING STARTED

Change all parts of Buddy's face given the following directions to create three more faces. Pay close attention to "width" and "length."

1. Godfrey's face is twice as wide and just as long as Buddy's face. Draw Godfrey's face.

2. Kilroy's face is twice as long and just as wide as Buddy's face. Draw Kilroy's face.

3. Dabney's face is twice as long and twice as wide as Buddy's face. Draw Dabney's face.



4. Look up <u>scale factor</u> in section 3.5, discuss it in class, and record it in the Word Bank.

5. Whose face represents Buddy's face scaled by a scale factor of 2? Dabney

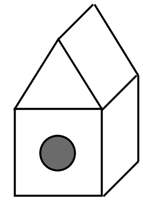
6. Whose face represents Dabney's face scaled by a scale factor of $\frac{1}{2}$? Buddy

7. Which two faces look the most alike? Buddy and Dabney
Be prepared to defend your opinion to your classmates.

A BIRD HOUSE

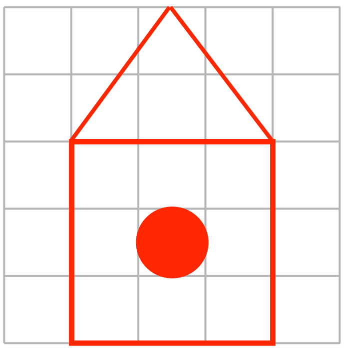
Lesson Notes

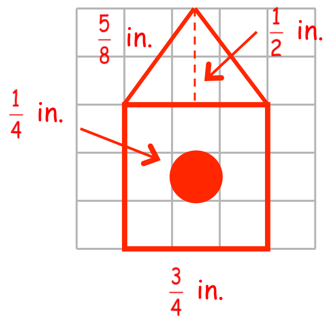
This is a birdhouse. Follow your teacher's directions to explore scale. Be sure to record this in your Word bank.

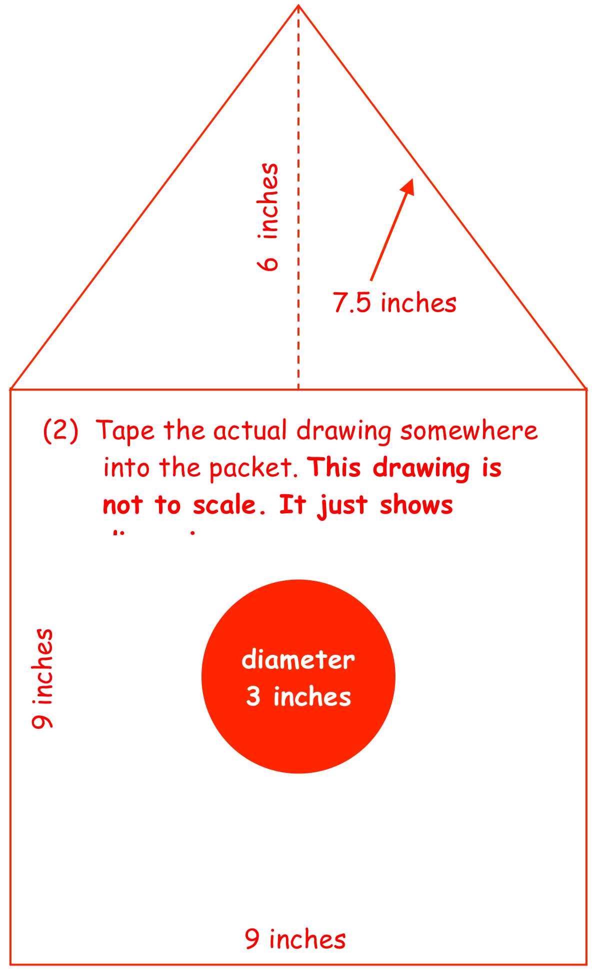


(1) Picture

(4) Tape the scale drawing into the packet.





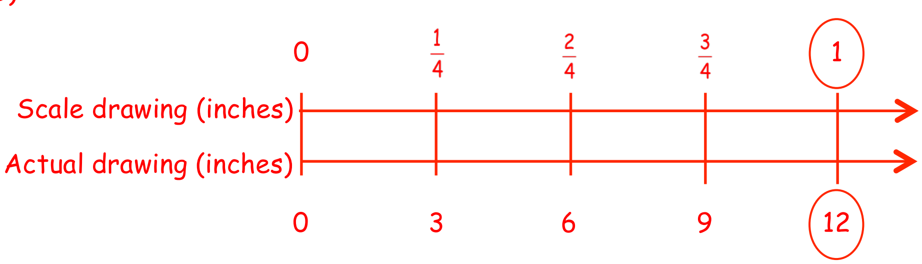


(2) Tape the actual drawing somewhere into the packet. **This drawing is not to scale. It just shows**

(3) Explanations will vary. For example:

- Since the length is 9 inches on the actual birdhouse, the length of each square on the picture must represent 3 inches.
- Since the length of the square on the picture represents 3 inches, the height of the triangle must be 6 inches.
- The circular opening must be 3 inches. I can use a compass with radius 1.5 inches to make it.
- Since the circle is centered in the square, I can fold the actual birdhouse square into quarters to locate the center of the circle and the apex of the triangle. OR I can find the center of the circle by drawing the diagonals in the square, and I can find the midpoint of the top side of the square and draw 6 inches up to find the apex of the circle.

(5)



1 inch on the drawing represents 12 inches on the actual drawing.

(6) The scale drawing is a reduction of the actual drawing. Its scale factor is $\frac{1}{12}$.

The actual drawing is an enlargement of the scale drawing. Its scale factor is 12.

(7) scale drawing : actual drawing → 1 : 12

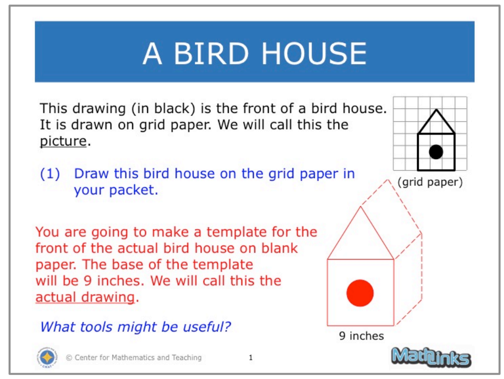
LESSON NOTES: A BIRD HOUSE

Note: Typically, we underline only key mathematical vocabulary. In this lesson, picture (in black on slides), scale drawing (in green on slides), and actual drawing (in red on slides) are underlined to distinguish among the various drawings of the birdhouse.

Materials to have available: rulers, compasses, large blank paper (11 x 17 works well, or two pieces of $8\frac{1}{2}$ x 11 taped together), $\frac{1}{4}$-inch graph paper, colored pencils, tape, scissors.

- Slide 1: For (1), students create a picture of the birdhouse in their packet.

  Suppose we wanted to make a actual drawing for the front of the bird house on blank paper so that we can cut it out of wood. What tools might be useful to make the actual drawing? Paper, rulers, scissors, tape, a compass to draw circle, etc.



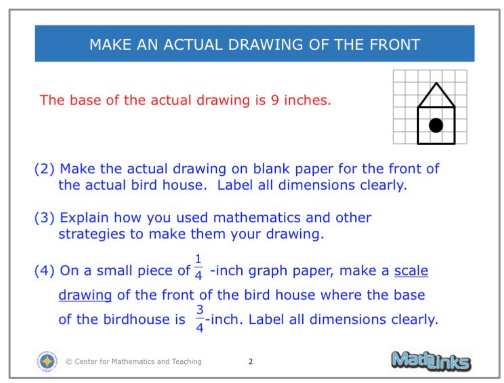
- Slide 2: For (2), Students make a actual drawing for the birdhouse on blank paper. Provide requested supplies, and give students time to create an accurate actual drawing and label dimensions on it.

  For (3), students explain in writing in their packets how they created two portions of the actual drawing.

  For (4), provide students with $\frac{1}{16}$ of a sheet of $\frac{1}{4}$-inch graph paper. Students create and cut out a scale drawing of the front of the birdhouse. This will be used during the discussion on Slide 3.

  If desired, students may decorate their scale drawing and actual drawing.

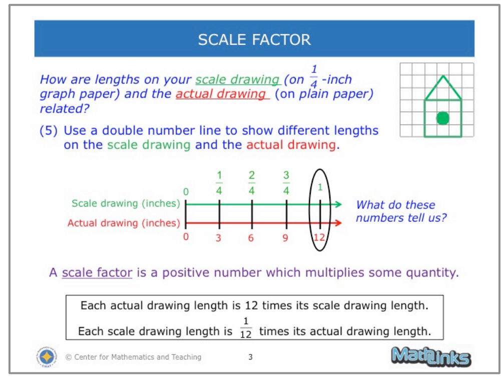
  Compare actual drawings and discuss strategies used.



For early finishers or to extend the activity, ask students to create scale drawings for all the faces of the bird house on $\frac{1}{4}$-inch graph paper. Challenge them to make a net and fold it to make a 3-dimensional model of the bird house. ***What dimension(s) are missing to complete this task?*** The depth of the birdhouse ***What would be a reasonable dimension?*** Answers will vary. If the floor of the birdhouse is a square, then the missing dimension would be 9 inches.

LESSON NOTES: A BIRD HOUSE

(Continued)

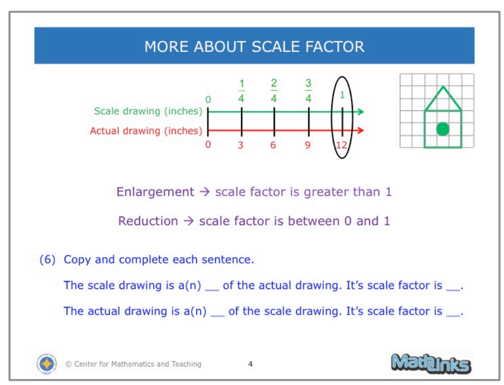


- Slide 3: Focus students on the relationships between corresponding lengths in the scale drawing and the actual drawing. ***How are lengths on the graph paper and the actual drawing related?*** Have students verify that 12 copies of the base of the scale drawing fit on the base of the actual drawing. ***Is this true for all dimensions?*** Yes, as long as drawings are accurate! ***How could we describe this relationship in words?*** Record student statements on the board. These can be revised after the definition of scale factor is revisited.

  For (5), ask students to create a double number line for dimensions of the scale drawing and actual drawing. Use the animated power point to scaffold as needed. These lengths represent a proportional relationship.

  Discuss the connection between scale and scale factor. Explain to students that scale factor is the value of a ratio (or rate). Since a double number line contains equivalent ratios, every pair of values on the double number line can be turned into a scale. A ratio of 12 : 1 results in a scale factor of 12. A ratio of 1 : 12 results in a scale factor of $\frac{1}{12}$.

  Revisit the student statements generated at the top of the slide, and revise them as needed. Two statements are provided on the slide.



- Slide 4: Explain the meaning of enlargement and reduction. ***Is the actual drawing an enlargement or reduction of the picture?*** Enlargement. ***Is the scale drawing an enlargement or reduction of the actual drawing?*** A reduction. ***How is the picture related to the scale drawing and actual drawing?*** It is an enlargement of the scale drawing and reduction of the actual drawing.

For (6), ask students to copy and complete the statements. Discuss as needed.

When completed, have students tape their scale drawing and actual actual drawing into their packets (or display some of them in the classroom).

LESSON NOTES: A BIRD HOUSE

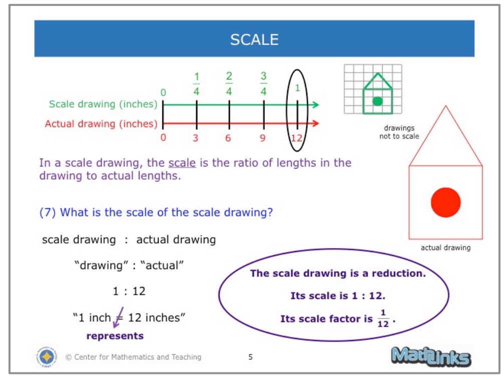
(Continued)

- Slide 5: Reveal the definition of <u>scale</u> and discuss.
  How are ratios and scales related? A scale is a ratio.

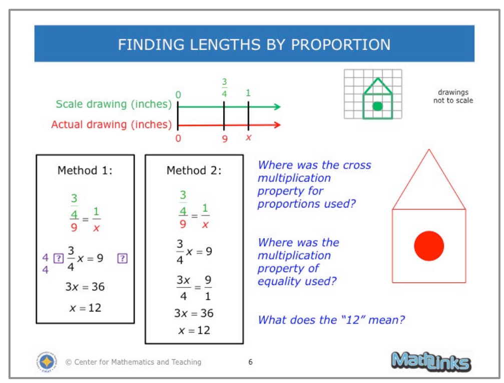
  For (7), ask students to write the scale for the birdhouse. Use the descriptions to help students write the ratio in the right order.

One way to make sure the order of the ratio is correct is to recognize whether the scale drawing is an enlargement or reduction. If the scale drawing is an enlargement, the value of the ratio will be greater than 1. If the scale drawing is a reduction, the value of the ratio will be between 0 and 1.

  Sometimes scales are written using an equal sign, such as "1 inch = 12 inches." What does this mean? It means that 1 inch on the scale drawing represents 12 inches on the actual figure. ***Even though it is commonly used, what is wrong with the statement mathematically?*** It is not a true equation. 1 inch is not the same as 12 inches.



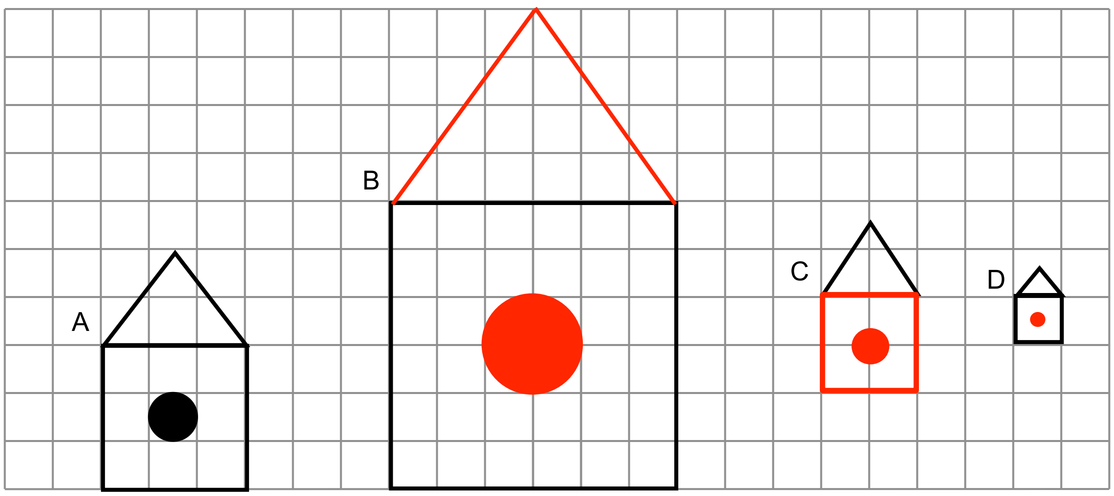
- Slide 6: If desired, use this animated slide to remind students how to use proportion as a mathematical strategy to find missing lengths on a scale drawing or an actual figure.



ENLARGEMENTS AND REDUCTIONS

Natasha is making scale drawings of a birdhouse she wants to build. She completed scale drawing A on graph paper for the front face of the birdhouse. Then she started drawings B, C, and D.

1. Complete drawings B, C, and D below.



2. Complete the table.

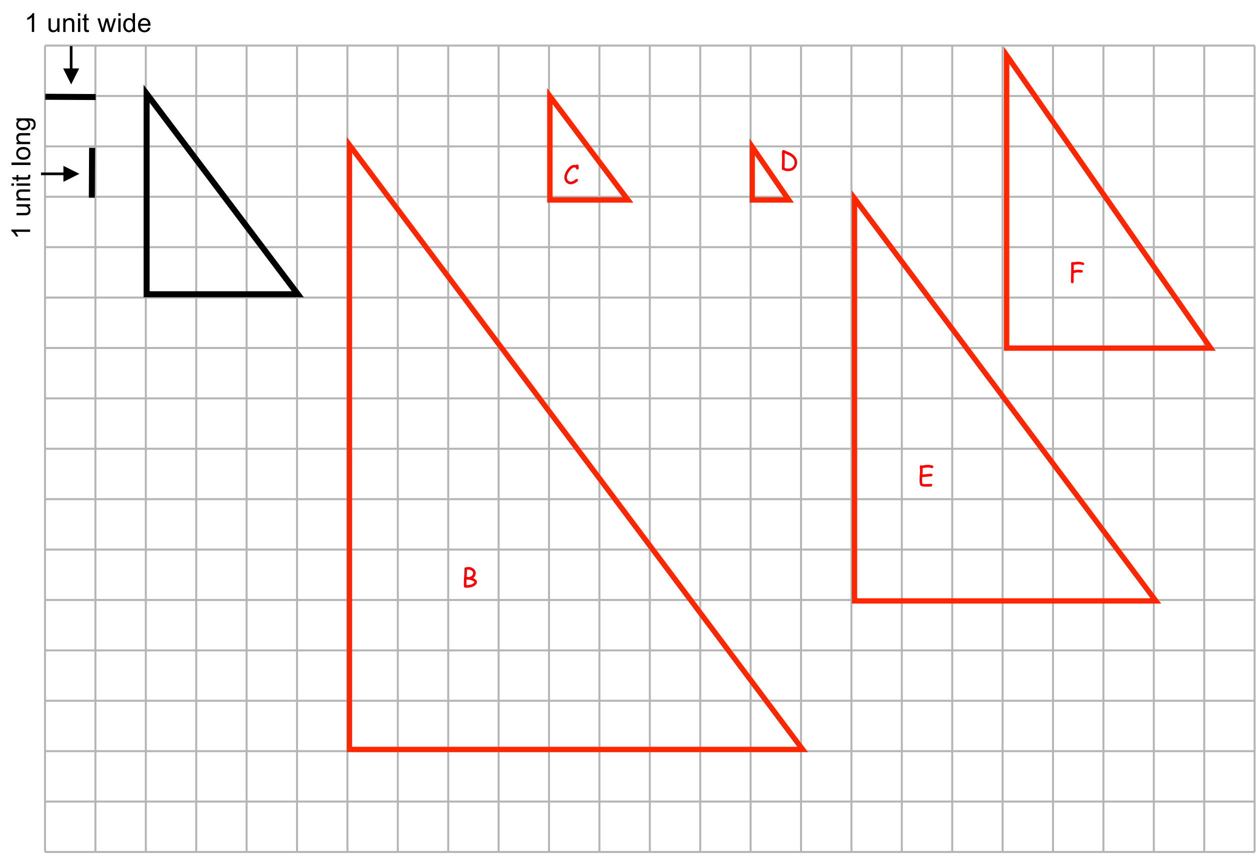
Drawing	Reduction or enlargement compared to drawing A?	Scale factor (multiplier) compared to drawing A	Scale (ratio) compared to drawing A
A	neither	1	1 : 1 (original size)
B	enlargement	2	2 : 1 (twice as large)
C	reduction	$\frac{2}{3}$	$\frac{2}{3}$: 1 (two-thirds as large)
D	reduction	$\frac{1}{3}$	$\frac{1}{3}$: 1 (one-third as large)

PRACTICE 7

Another way to describe scale factor is as a percent. For example, a scale factor of 2 could also be described as a scale factor of 200%.

Based on triangle A, complete the table and draw each triangle on the grid paper below.

	Compared to Triangle A					
Triangle	**Scale Factor (as a percent)**	**Scale factor (as a number)**	**Scale (ratio)**	**Enlargement or Reduction**	**Height** (length) "long"	**Base** (width) "wide"
A	100%	1	1 : 1	neither	4 units	3 units
B	300%	3	3 : 1	enlargement	12 units	9 units
C	50%	$0.5 = \frac{1}{2}$	0.5 : 1 or 1 : 2	reduction	2 units	$1\frac{1}{2}$ units
D	25%	$0.25 = \frac{1}{4}$	0.25 : 1 or 1 : 4	reduction	1 unit	$\frac{3}{4}$ units
E	200%	2	2 : 1	enlargement	8 units	6 units
F	150%	$1.5 = 1\frac{1}{2}$	1.5 : 1 or 3 : 2	enlargement	6 units	$4\frac{1}{2}$ units



REVIEW

POSTER PROBLEMS: IT'S ABOUT TIME!

Directions for Poster Problems are located in the Teaching Tips in PR1.

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions. Use a calculator as needed.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
A watch gains 2 minutes in 6 hours.	Mary read 22 pages in 30 minutes.	Betsy cooks 17 hours in a 2-week period.	Hurricane Katrina dropped 14 inches of rain over a 48-hour period.

A. Copy the fact statement. Find a simple unit rate to describe the situation (This rate may use a different unit of time.) Unit rates may vary.
(1) 8 min/day (2) 44 pgs/hr (3) 34 hrs/month OR 8.5 hrs/wk (4) 7 inches/day

B. Assume a proportional relationship. Make a double number line that compares the quantities for different reasonable amounts of time. Double number lines will vary based upon unit rates used.

C. Write a question that can be answered using the fact statement. Questions will vary.

D. Answer the question asked in part C. Answers based upon questions created.

Part 3: Return to your seats. Work in partners or groups.

1. Look back at each fact statement. Is it reasonable for this relationship to hold up over an extended period of time? Explain.

   (1) Yes. The watch is broken.
   (2) No. She will do other activities, like eat and sleep.
   (3) Possibly, if it is a pattern.
   (4) No. The storm ended.

2. If a watch gains 2 minutes every 6 hours, what time is it?
   Possible answers: Not enough information to answer the question.
   Time to get your watch fixed.

R

MATCHING ACTIVITY: NUTS

1. Your teacher will give you some cards that represent proportional relationships, except for one error on one card. Work with a partner to match cards with equivalent representations and find the error. If desired, have groups of students divide up the equivalent cards and attach some of them in their packet.

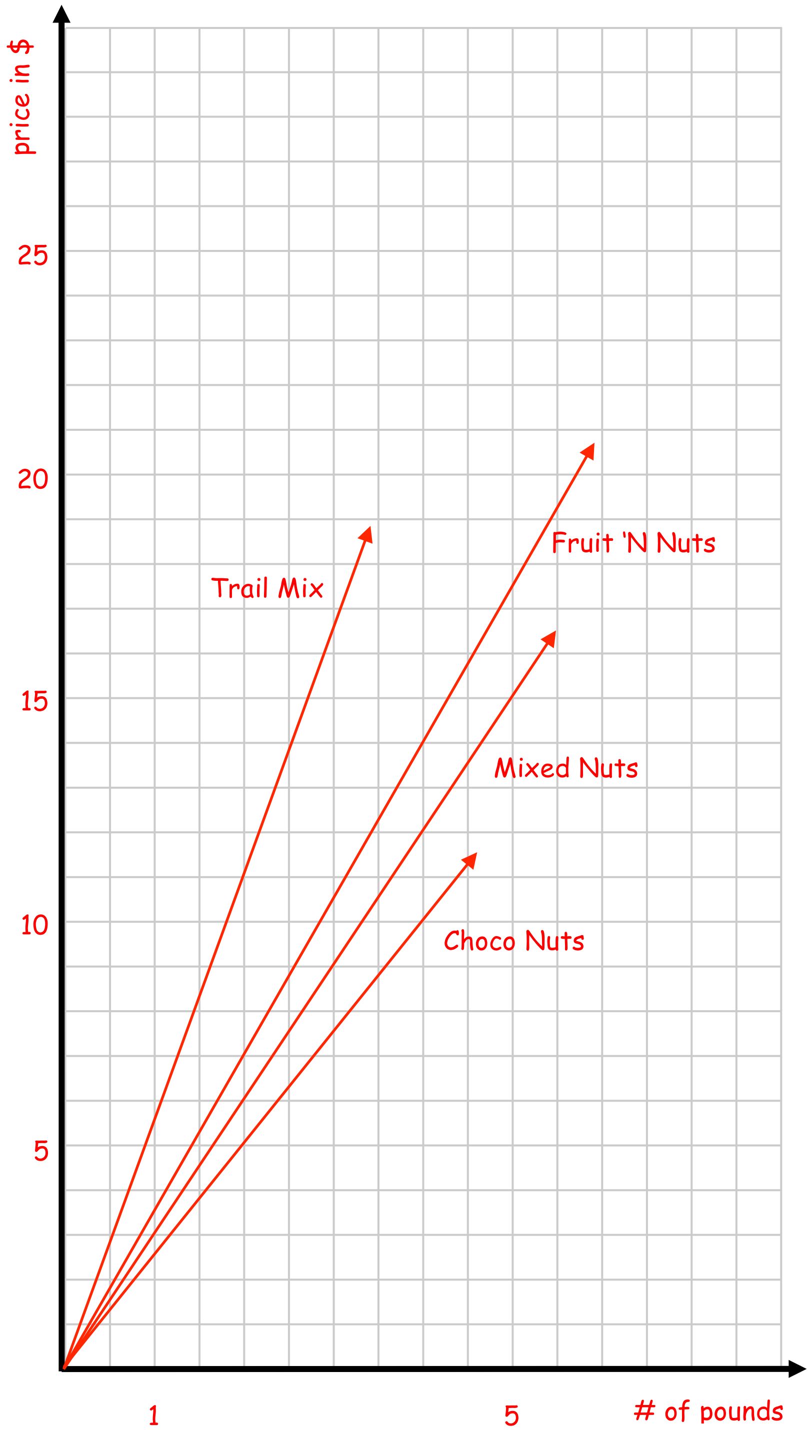
2. What was the error? How do you know? Fix it on the card.

   For Mixed Nuts, 4 pounds should cost $12. Since it is supposed to be a proportional relationship the cost should be $3 per pound.

3. Graph the cost vs. quantity for each mixture on the graph using different colors.

4. Do you think the points should be connected? Explain.

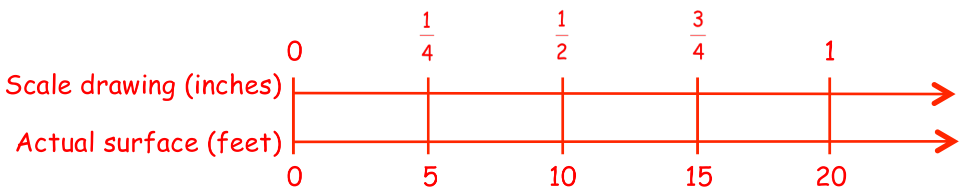
   Students might make the case either way. Nuts sold by the pound can be purchased in any quantity, so you can make the case that it makes sense to connect the points. However, we don't purchase in fractions of a penny.



SPORTS PLAYING SURFACES

You will make scale drawings of sports playing surfaces.

1. Draw a double number line that shows a scale of $\frac{1}{2}$ in. : 10 ft.



Determine the dimensions of each sports surface if the scale is $\frac{1}{2}$ in. : 10 ft. You may want to use the double number line to help you.

	Sport Surface	Actual Length	Actual Width	Drawing length	Drawing width
2.	Soccer Field	400 ft	300 ft	20 inches	15 inches
3.	Volleyball Court	60 ft	30 ft	3 inches	$1\frac{1}{2}$ inches
4.	Football Field	360 ft	160 ft	18 inches	8 inches
5.	Roller Rink	70 ft	150 ft	$3\frac{1}{2}$ inches	$7\frac{1}{2}$ inches
6.	Bowling Lane	60 ft	4 ft	3 inches	$\frac{1}{5}$ inch
7	(Your choice – research on internet)				

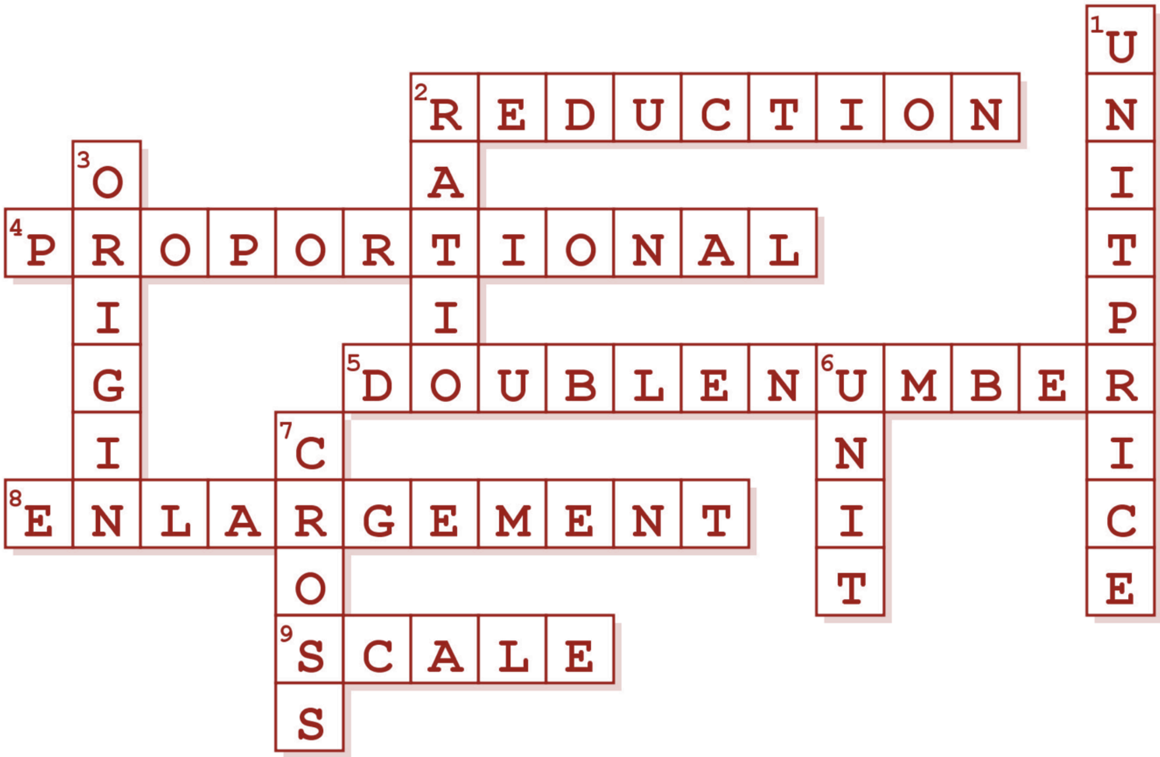
8. **Project**: Use tools of your choice. Choose two of the sports surfaces above and create scale drawings. You may want to research other features on the internet to include on your scale drawings. Cut them out and label completely.

   I made scale drawings for a ____________________ and a ____________________.

9. Use your drawings and explain approximately how many copies of your smaller sports surface will fit inside your larger sports surface.

   Answers will vary. For example, 7.5 bowling lanes will fit in a volleyball court.

FOCUS ON VOCABULARY



Across

2 result of a scale factor between 0 and 1

4 the graph of a ____ relationship is a straight line through the origin

5 ______ lines are parallel lines used to show a proportional relationship (two words)

8 result of a scale factor greater than 1

9 a multiplier; ______ factor

Down

1 cost for 1 (two words)

2 comparison of two numbers

3 the point (0,0)

6 rate for 1; ______ rate

7 a strategy for solving proportions; ______ multiplication property

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
cross-multiplication property	The cross-multiplication property for proportions states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. From $\frac{2}{3} = \frac{8}{12}$ we have $3 \bullet 8 = 2 \bullet 12$.
proportion	A proportion is an equation stating that the values of two ratios are equal. The equation $\frac{3}{25} = \frac{12}{100}$ is a proportion. It asserts that the values of the ratios 3 : 25 and 12 : 100 are equal. The value of both ratios is 0.12.
proportional	Two variables are proportional if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a proportional relationship, and the constant is referred to as the constant of proportionality. If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.
scale	In a scale drawing of a figure, the scale is the ratio of lengths in the drawing to lengths in the figure. A blueprint of a house floorplan has a scale of 1 inch to 5 feet, or 1 in : 5 ft. Each inch on the blueprint represents 5 feet in the actual house. A drawing of a lady bug has a scale of 5 cm to 2 millimeters, or 5 cm : 2 mm. Each 5 cm on the drawing represents 2 mm on the actual bug.

Word or Phrase	Definition
scale drawing	A scale drawing of a geometric figure is a drawing in which all lengths have been multiplied by the same scale factor, while angles remain the same. A blueprint of a house floorplan is a scale drawing.
scale factor	A scale factor is a positive number which multiplies some quantity. To make a scale drawing of a figure, we multiply all lengths by the same scale factor, keeping all angles equal to those in the original figure. If the scale factor is greater than 1, the scale drawing is an enlargement of the actual figure. If the scale factor is between 0 and 1, the scale drawing is a reduction of the actual figure. A drawing of a ladybug has a scale of 5 cm : 2 mm. This is equivalent to 50 mm : 2 mm. The scale factor is $\frac{50}{2}$ = 25. The drawing is an enlargement. A blueprint of a house floorplan has a scale of 1 in : 5 ft. This is equivalent to 1 in : 60 in. The scale factor is $\frac{1}{60}$. The blueprint is a reduction.
unit price	A unit price is a price for one unit of measure.
unit rate	The unit rate associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. The ratio of 40 miles each 5 hours has unit rate 8 miles per hour.
value of a ratio	The value of the ratio $a : b$ is the number $\frac{a}{b}$, $b \neq 0$. The value of the ratio 6 : 2 is $\frac{6}{2}$ = 3. The value of the ratio of 3 to 2 is $\frac{3}{2}$ = 1.5.

Sense-Making Strategies to Solve Proportional Reasoning Problems

How much will 5 pencils cost if 8 pencils cost $4.40?

Strategy 1: Use a "halving" strategy

If 8 pencils cost $4.40, then
4 pencils cost $2.20,
2 pencils cost $1.10, and
1 pencil costs $0.55.

Therefore, 5 pencils cost

$0.55 + $2.20 = $2.75.

Strategy 2: Find unit prices

First, find the cost of one pencil.

$$\frac{\$4.40}{8} = \$0.55$$

Then, multiply by 5 to find the cost of 5 pencils,

($0.55)(5) = $2.75.

Sammie can crawl 12 feet in 3 seconds. At this rate, how far can she crawl in $1\frac{1}{2}$ minutes?

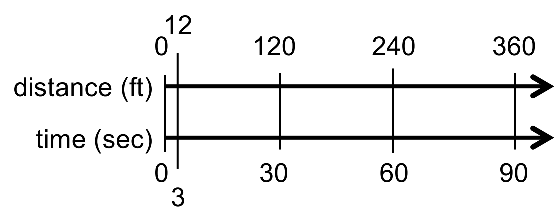
Strategy 1: Make a table

Distance	Time
12 ft	3 seconds
4 ft	1 second
240 ft	60 sec = 1 min
120 ft	30 sec = $\frac{1}{2}$ min
360 ft	90 sec = $1\frac{1}{2}$ min

Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Strategy 2: Make a Double Number Line

$1\frac{1}{2}$ minutes = 90 seconds.



Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

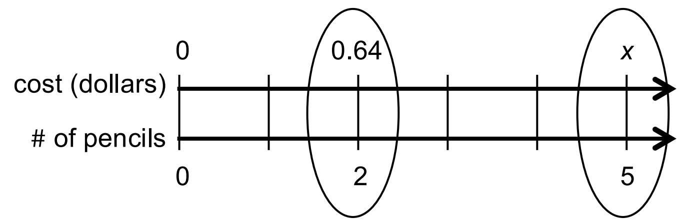
Setting Up Proportions

Here are some ways to set up a proportion to solve a problem.

If 2 pencils cost \$0.64, how much will 5 pencils cost?

Strategy 1: Compare rates. This is sometimes referred to as a "between" proportion because the ratios contain different units (i.e., between two units). Between proportions can be read directly from a double number line.

Organize known information on a double number line if desired. From the diagram we see:



$$\frac{0.64 \text{ dollars}}{2 \text{ pencils}} = \frac{0.64}{2}\ \frac{\text{dollar}}{\text{pencil}} \qquad \frac{x \text{ dollars}}{5 \text{ pencils}} = \frac{x}{5}\ \frac{\text{dollar}}{\text{pencil}}$$

Then, equate the two expressions and solve for x. $\frac{x}{5} = \frac{0.64}{2}$

$x = 1.60$ dollars for 5 pencils.

Note: The equation $\frac{5}{x} = \frac{2}{0.64}$ is another valid "between" proportion for this problem.

Strategy 2: Compare like units. This is sometimes referred to as a "within" proportion because the ratios contain the same units (i.e., within the same unit).

Use a double number line to organize information if you wish.

Create one ratio based on corresponding costs and another ratio based on the corresponding numbers of pencils.

$$\frac{\text{cost}_{\text{case 1}}}{\text{cost}_{\text{case 2}}} = \frac{0.64}{x} \qquad \frac{\text{pencils}_{\text{case 1}}}{\text{pencils}_{\text{case 2}}} = \frac{2}{5}$$

Then, equate the two ratios, and solve for x. $\frac{0.64}{x} = \frac{2}{5}$

$x = 1.60$ dollars for 5 pencils.

Note: The equation $\frac{x}{0.64} = \frac{5}{2}$ is another valid "within" proportion for this problem.

Some Properties Relevant to Solving Proportions

Here are some important properties of arithmetic and equality related to proportions.

- The multiplication property of equality states that equals multiplied by equals are equal.
  Thus, if $a = b$ and $c = d$, then $ac = bd$.

  Example: If $\frac{6}{2} = 3$ and $5 = 9 - 4$, then $\frac{6}{2}(5) = 3(9 - 4)$.

- The fraction-inverse property for proportions states that if two nonzero fractions are equal, then their inverses are equal. That is, if $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ ($a \neq 0$, $b \neq 0$, $c \neq 0$, $d \neq 0$).

  Example: If $\frac{5}{7} = \frac{12}{x}$, then $\frac{7}{5} = \frac{x}{12}$

- The cross-multiplication property for proportions states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ ($b \neq 0$, $d \neq 0$).

  This can be remembered with the diagram: $\frac{a}{b} = \frac{c}{d}$.

  Example: If $\frac{5}{7} = \frac{12}{x}$, then $5 \bullet x = 7 \bullet 12$.

  To see that this property is reasonable, try simple numbers:

  If $\frac{3}{4} = \frac{6}{8}$, then $3 \bullet 8 = 4 \bullet 6$.

Applying Properties to Solve Proportions

Strategy 1:
Multiplication Property of Equality

Solve for x:

Multiplication Property of Equality

$$\frac{x}{12} = \frac{3}{8}$$

$$(8 \bullet \cancel{12}) \bullet \frac{x}{\cancel{12}} = \frac{3}{\cancel{8}} \bullet (\cancel{8} \bullet 12)$$

$$8x = 36$$

$$x = \frac{36}{8}$$

$$x = 4\frac{1}{2}$$

Strategy 2:
Cross-Multiplication Property

Solve for x:

Cross-multiplication property

$$\frac{x}{12} = \frac{3}{8}$$

$$8 \bullet x = 3 \bullet 12$$

$$8x = 36$$

$$x = \frac{36}{8}$$

$$x = 4\frac{1}{2}$$

Testing for a Proportional Relationship

Here are three ways to test if two variables are in a proportional relationship:

- The values of the ratios (unit rates or unit prices) created by data pairs are the same.
- An equation in the form $y = kx$ fits all corresponding data pairs.
- Graphed data pairs fall on a line through the origin (0, 0).

Alexa buys tickets when she goes to the amusement park. This chart shows the costs for different quantities of tickets.

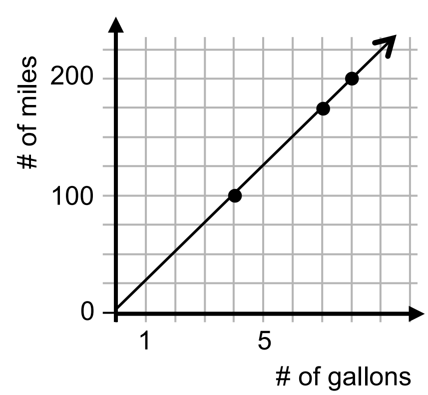
# of tickets	10	20	25	50	100
total cost	\$40	\$60	\$75	\$125	\$200
cost per ticket	\$4	\$3	\$3	\$2.50	\$2

Since the costs per ticket (unit prices) are not the same, ticket purchasing at this amusement park does not represent a proportional relationship.

Antonio kept track of the number of miles he traveled each time he filled his tank with gas. Here is some data.

number of miles	100	200	175	300
number of gallons	4	8	7	12
miles per gallon	25	25	25	25

Since the miles per gallon (unit rates) created by the data pairs is the same, this situation represents quantities in a proportional relationship.



Furthermore,

Let x = the number of gallons
Let y = the number of miles

The data fits the equation $y = 25x$ (an equation in the form $y = kx$), which is an equation that represents a proportional relationship.

Finally, if the points for (gallons, miles) are graphed, they will fall on a line through the origin (0,0).

Multiple Representations and Proportional Relationships

Suppose 4 balloons cost $6.00 and each balloon is the same price. Here are some strategies for representing this proportional relationship.

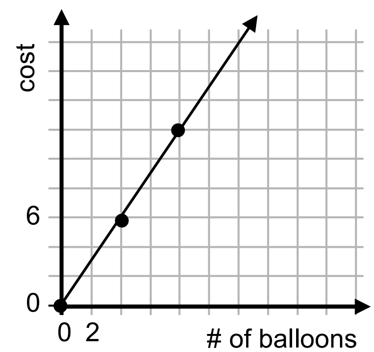
Strategy 1: Tables

Create a table to calculate unit rates. If the unit rates are the same, the variables are in a proportional relationship.

Number of Balloons	Cost	Unit Price
4	$6.00	$1.50
2	$3.00	$1.50
1	$1.50	$1.50
8	$12.00	$1.50

Strategy 2: Graphs

A straight line through the origin indicates quantities in a proportional relationship.



Strategy 3: Equations

An equation of the form $y = kx$ indicates quantities in a proportional relationship. In this case,

y = cost in dollars
x = number of balloons
k = cost per balloon (unit price)

To determine the unit price, create a ratio whose value is: $\dfrac{6 \text{ dollars}}{4 \text{ balloons}} = 1.50 \dfrac{\text{dollars}}{\text{balloon}}$

Therefore, k = 1.50 dollars per balloon, and

$$y = 1.50x.$$

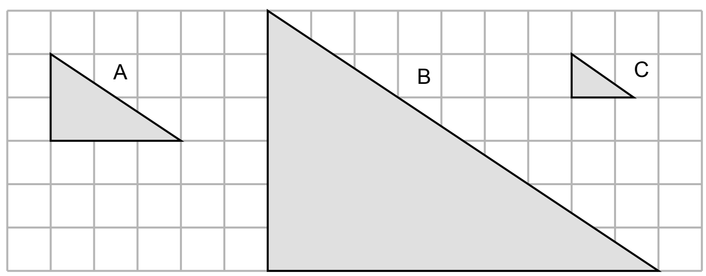
This equation expresses the output as a constant multiple of the input, showing that the relationship is proportional.

Scale Drawings

A scale drawing of a geometric figure is a drawing in which all distances have been multiplied by the same scale factor, while angles remain the same. If the scale factor is greater than 1, the figure is expanded (enlarged), and if the scale factor is between 0 and 1, the figure is reduced in size.

The ratio of lengths in the drawing to lengths in the actual figure is the scale of the drawing.
A scale of 1:1 implies that the drawing is the same as the actual object. A scale 1 : 2 implies that the drawing is smaller (half the size) than the actual object (in other words, the dimensions are multiplied by a scale factor of 0.5).

- To make Triangle B below, multiply each dimension of Triangle A by a scale factor of 3. Triangle B is a 300% enlargement of Triangle A. The scale is 1 : 3.
- To make Triangle C below, multiply each dimension of Triangle A by a scale factor of $\frac{1}{2}$. Triangle C is a 50% reduction of triangle A. The scale is 2 : 1.



Scale Drawing of a Flag

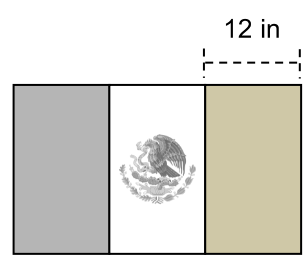
A scale drawing of a geometric figure is a drawing in which all distances have been multiplied by the same scale factor while angles remain the same. If the scale factor is greater than 1, the figure is expanded (enlarged), and if the scale factor is between 0 and 1, the figure is reduced in size.

The flag of Mexico is composed of three stripes (green, white, and red) that divide the flag into thirds. The national coat of arms is in the center of the white stripe. Pictured below is a (gray) scale drawing of the flag.

Suppose the original flag is 36 inches by 24 inches, and the scale drawing is 1.5 inches by 1 inch.

This scale may be represented as a ratio:

scale drawing	:	actual flag
1.5 inch	:	36 inches
1 inch	:	24 inches
1	:	24



The scale drawing is a reduction of the flag, with scale factor (value of the ratio) $\frac{1}{24}$.

The ratio "1 inch on the scale drawing represents 24 inches on the actual flag" is sometimes written with the technically incorrect, but convenient notation "1 inch = 24 inches." We will not do this because everyone knows that 1 inch does not really equal 24 inches!

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